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Non-uniform double slot injection (suction) on a forced flow over a slender cylinder

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Abstract

A general analysis has been developed to investigate the influence of non-uniform double slot injection (suction) on the steady non-similar incompressible laminar boundary layer flow over a slender cylinder, where the slender cylinder is inline with the flow. Non-similar solutions are obtained starting from the origin of the stream-wise coordinate along the stream-wise direction by using an implicit finite difference scheme in combination with the quasilinearization technique. Numerical results are reported to display the effects of non-uniform double slot injection/suction on skin friction coefficient and heat transfer rate at the wall. Further, the effects of viscous dissipation and Prandtl number on velocity and temperature profile, and skin friction and heat transfer co-efficients are also presented in this paper.

Keywords: Slender cylinder; Double slot injection (suction)

1. Introduction

In recent years, the flow and heat transfer phenomena over slender cylinders have received a considerable attention due to its practical needs because the use of a slender body reduces the drag and even produces sufficient lift to support the body in certain situations. In such a case, the governing equations contain the transverse curvature term which strongly influences the velocity and temperature fields and correspondingly the skin friction coefficient and heat transfer rate at the wall [1,2]. Recently, Takhar et al. [3] have studied the combined effect of free and forced convection flows over a vertical slender cylinder. The calculations of momentum and heat transfer on slender cylinders should consider the transverse curvature effect, especially in applications such as wire and fiber drawing, where accurate predictions are required and thick boundary layers can exist on slender or near-slender bodies. In particular, the multi-spinning process consists of extruding a molten polymer from an array of holes in a spinneret. The yarns that are created are cooled down by their motion in the air and by a lateral quenching air flux, while their diameter decreases because of stretching provided by the rotating bobbin, until they solidify. In these applications, the careful control of yarn-quenching temperature or the heating and cooling temperature has a strong bearing of final product quality [4].

Mass transfer through a slot strongly influences the development of a boundary layer along a surface and can prevent or at least delay the separation of the viscous region. Several investigators [5,6] have studied the effect of single slot injection (suction) into steady compressible and water boundary layer flows over two dimensional and

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Nomenclature

A	surface mass transfer parameter
C_f	local skin friction coefficient
C_p	specific heat at constant pressure
Ec	Eckert number
f, F	dimensionless stream function, velocity component, respectively
G	dimensionless temperature
k	thermal conductivity
Nu	local Nusselt number
Pr	Prandtl number
r	radial coordinate
Re_x	Reynolds number
r_0	radius of cylinder
T	temperature
U	free stream velocity component
u, v	axial and radial velocity components, respectively
x	axial coordinate
Greek symbols	
η	similarity variable
μ, ν	dynamic and kinematic viscosities, respectively
ξ	transverse curvature
ω^*	slot length parameter
ρ	density
Subscripts	
w, ∞	conditions at the wall and infinity, respectively
ξ, η	denote the partial derivatives w.r.t to these variables, respectively

axi-symmetric bodies. Moreover, Roy [7] and Subhashini et al. [8] have investigated the influence of non-uniform double slot injection (suction) on compressible boundary layer flows over cylinders, spheres and yawed cylinders, respectively. Also in a recent study, Datta et al. [9] have reported the influence of non-uniform single slot injection (suction) on an incompressible boundary layer flow over a slender cylinder including the effect of transverse curvature. Therefore, as a step towards the eventual development in the study of mass transfer into the boundary layer flows, it is interesting as well as useful to investigate the combined effects of non-uniform double slot injection (suction), viscous dissipation, thermal diffusion and transverse curvature on a slender cylinder.

The objective of the present investigation is to study the influence of non-uniform double slot injection (suction) on an incompressible boundary layer flow over a slender cylinder including the effects of transverse curvature, viscous dissipation and thermal diffusion. There are two types of free parameters in this problem, one type of parameter measures the length of the slots (i.e., the parts of the body surface in which there is mass transfer) and another type of parameter fixes the position of the slots. Thus, these two sets of parameters help to vary the lengths and locations of the slots, respectively.

2. Analysis

We consider the steady laminar forced convection flow over a slender cylinder of radius r_0 with non-uniform slot injection/suction. The flow is taken to be axisymmetric and Fig. 1 shows the co-ordinate system and the physical model. The blowing rate is assumed to be small and it does not affect the invicid flow at the edge of the boundary layer [10]. The effects of transverse curvature and viscous dissipation are also included in the analysis. The fluid at the edge of the boundary layer is maintained at a constant temperature T_∞ and the body has a uniform temperature T_w ($T_w >$ or $< T_\infty$, i.e. the slender cylinder is either heated or cooled). It is assumed that the injected fluid posses the same physical properties as the boundary layer fluid and has a static temperature equal to the wall temperature. Under the above assumptions, the governing boundary layer momentum and energy equations can be expressed in non-dimensional form as [9]

$$(1+\xi\eta)F_{\eta\eta} + (\xi + f)F_\eta = \xi(FF_\xi - F_\eta f_\xi), \tag{1}$$

$$Pr^{-1}(1+\xi\eta)G_{\eta\eta} + (\xi Pr^{-1} + f)G_\eta + Ec(1+\xi\eta)F_\eta^2 = \xi(FG_\xi - G_\eta f_\xi), \tag{2}$$

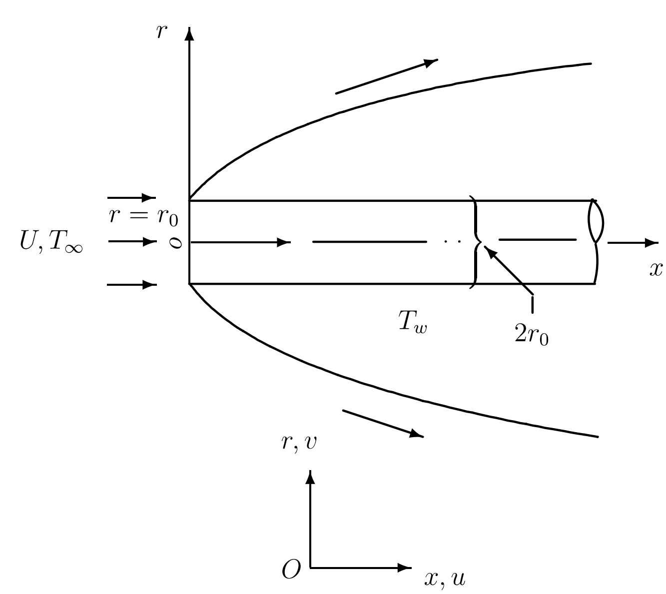

Fig. 1. Physical model and coordinate system.

where

$$\xi = \left(\frac{4}{r_0}\right)\left(\frac{\nu x}{U}\right)^{\frac{1}{2}}, \quad \eta = \left(\frac{U}{\nu x}\right)^{\frac{1}{2}}\left[\frac{r^2 - r_0^2}{4r_0}\right],$$
$$\frac{r^2}{r_0^2} = [1+\xi\eta], \quad \psi(x,r) = r_0(\nu U x)^{\frac{1}{2}} f(\xi,\eta),$$
$$u = \frac{1}{2}Uf_\eta = \frac{U}{2}F, \quad v = \frac{1}{2r}r_0\left(\frac{\nu U}{x}\right)^{\frac{1}{2}}(\eta f_\eta - f - \xi f_\xi), \tag{3}$$
$$f_\eta(\xi,\eta) = F(\xi,\eta), \quad G(\xi,\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$
$$Ec = \frac{U^2}{4C_p(T_w - T_\infty)}, \quad Pr = \frac{\mu C_p}{k}.$$

The transformed boundary conditions on the set of Eqs. (1) and (2) are

$$\begin{aligned} &F(\xi,0) = 0, \quad G(\xi,0) = 1 \quad \text{at } \eta = 0, \\ &F(\xi,\infty) = 2, \quad G(\xi,\infty) = 0 \quad \text{as } \eta \to \infty, \end{aligned} \tag{4}$$

where $f = \int_0^\eta F\,\mathrm{d}\eta + f_w$ and f_w is given by

$$f_w = -\left(\frac{r_0}{2\nu\xi}\right)\int_0^\xi \xi v_w(\xi)\mathrm{d}\xi,$$
$$v_w(\xi) = -\left(\frac{\nu U}{x}\right)^{\frac{1}{2}}\frac{1}{2}[f + \xi f_\xi] = -\frac{2\nu}{r_0\xi}[f + \xi f_\xi].$$

Here the boundary condition $v_w(x)$ is considered in terms of transformed coordinate ξ and $v_w(\xi)$ is taken as sinusoidal function given by

$$\begin{aligned} v_w(\xi) &= -A\left(\frac{2\nu}{r_0}\right)\omega^*\sin[\omega^*(\xi - \xi_1)], \quad \xi_1 \leqslant \xi \leqslant \xi_1^*, \\ &= -A\left(\frac{2\nu}{r_0}\right)\omega^*\sin[\omega^*(\xi - \xi_2)], \quad \xi_2 \leqslant \xi \leqslant \xi_2^*, \\ &= 0, \quad \text{otherwise}. \end{aligned}$$

Therefore f_w can be written as

$$\begin{aligned} f_w &= 0 \quad \text{for } \xi \leqslant \xi_1, \\ &= \left(\frac{A}{\xi}\right)\phi(\xi_1,\xi) \quad \text{for } \xi_1 \leqslant \xi \leqslant \xi_1^*, \\ &= \left(\frac{A}{\xi}\right)\phi(\xi_1,\xi_1^*) \quad \text{for } \xi_1^* \leqslant \xi \leqslant \xi_2, \\ &= \left(\frac{A}{\xi}\right)\phi(\xi_1,\xi_1^*) + \left(\frac{A}{\xi}\right)\phi(\xi_2,\xi) \quad \text{for } \xi_2 \leqslant \xi \leqslant \xi_2^*, \\ &= \left(\frac{A}{\xi}\right)\phi(\xi_1,\xi_1^*) + \left(\frac{A}{\xi}\right)\phi(\xi_2,\xi_2^*) \quad \text{for } \xi \geqslant \xi_2^*, \end{aligned}$$

where $\phi(s,t) = s - t\cos\{\omega^*(t-s)\} + \frac{1}{\omega^*}\sin\{\omega^*(t-s)\}$.

Here ω^* and $\xi_1, \xi_2 (0 < \xi_1 < \xi_1^* < \xi_2 < \xi_2^*)$ are two sets of free parameters which, respectively, determine the lengths and locations of the slots. The subscripts '1' and '2' denote the first and second slots, respectively. The function $v_w(\xi)$ is continuous for all values of ξ and it has non-zero values only in the intervals (ξ_1, ξ_1^*) and (ξ_2, ξ_2^*). The surface mass transfer parameter $A > 0$ or $A < 0$, according to whether there is a suction or injection. The local skin friction coefficient and heat transfer rate are given by

$$C_\mathrm{f} = \frac{2\left[\mu\frac{\partial u}{\partial r}\right]_w}{\rho U^2} = 2^{-1}(Re_x)^{-\frac{1}{2}}(F_\eta)_w \text{ and } Re_x^{-\frac{1}{2}}Nu = -2^{-1}(G_\eta)_w$$

where

$$Re_x^{\frac{1}{2}}C_\mathrm{f} = 2^{-1}(F_\eta)_w \text{ and } Nu = -\frac{\left[x\left(\frac{\partial T}{\partial r}\right)\right]_w}{T_w - T_\infty}$$

3. Results and discussion

The boundary value problem represented by Eqs. (1) and (2) along with the boundary conditions (4) is solved by implicit finite difference scheme in combination with quasi-linearization technique. Since the method is described in complete details in [6,11], it is detailed description is not presented here. To ensure the convergence of the numerical solution to exact solution, the step sizes $\Delta\eta$ and $\Delta\xi$ have been optimized and taken as $\Delta\xi = 0.01$ and $\Delta\eta = 0.02$ throughout the computations. A convergence criteria based on the relative difference between the current and previous iteration values of the velocity and temperature gradients at wall are employed. When the difference reaches less than 10^{-4}, the solution is assumed to have converged and the iterative process is terminated. Computations have been carried out for various values of $Pr(0.7 \leqslant Pr \leqslant 7.0)$, $Ec(-0.25 \leqslant Ec \leqslant 0.3)$ and $A(-0.6 \leqslant A \leqslant 0.4)$. In order to validate our method, we have compared results of skin friction and heat transfer parameters $(F_\eta(0,0),\ G_\eta(0,0))$ with those of Takhar et al. [3]. Comparison of skin friction and heat transfer coefficients is also made with the most recent results of Datta et al. [9] for the non-uniform single slot injection/suction. Earlier results are found to be in excellent agreement with the present results and comparisons are not shown here for the sake brevity.

Figs. 2–5 show the effects of non-uniform double slot injection (or suction) parameter ($A < 0$ or $A > 0$) and ξ_1,

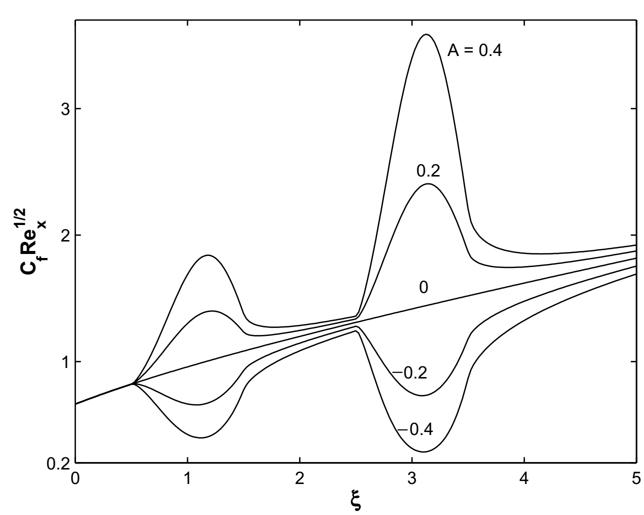

Fig. 2. Effects of suction ($A > 0$) and injection ($A < 0$) on $C_\mathrm{f}(Re_x)^{1/2}$ when $Ec = 0.1$, $Pr = 0.7$, $\xi_1 = 0.5$, $\xi_2 = 2.5$ and $\omega^* = \pi$.

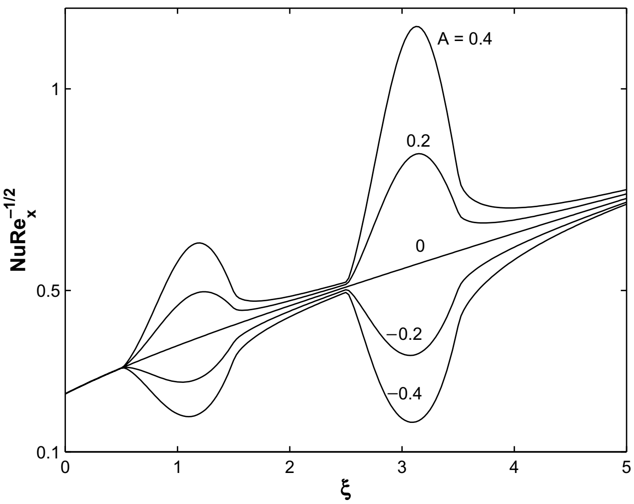

Fig. 3. Effects of suction ($A > 0$) and injection ($A < 0$) on $Nu(Re_x)^{-1/2}$ when $Ec = 0.1$, $Pr = 0.7$, $\xi_1 = 0.5$, $\xi_2 = 2.5$ and $\omega^* = \pi$.

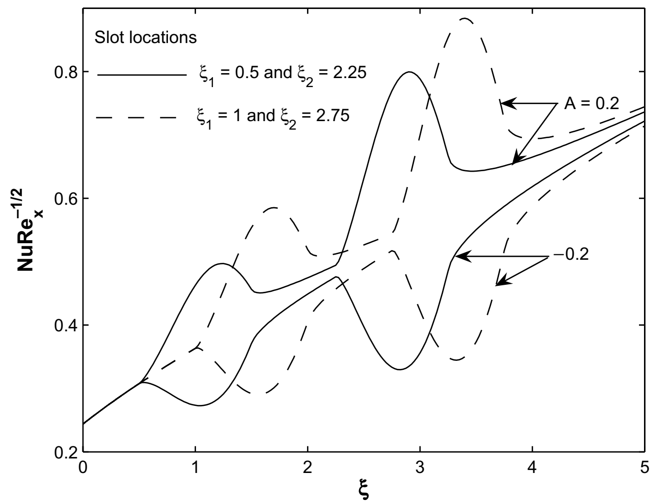

Fig. 5. Slot positions's variation effect on $Nu(Re_x)^{-1/2}$ when $Ec = 0.1$, $Pr = 0.7$ and $\omega^* = \pi$.

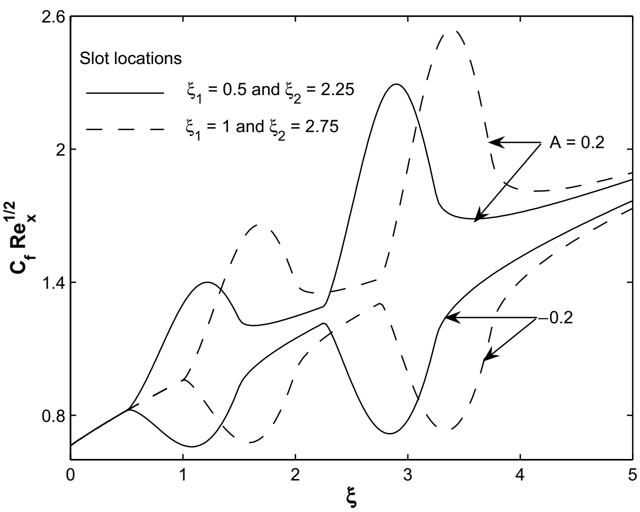

Fig. 4. Slot positions's variation effect on $C_f(Re_x)^{1/2}$ when $Ec = 0.1$, $Pr = 0.7$ and $\omega^* = \pi$.

ξ_2, which fix the slots positions (i.e., the porous sections on the surface of the body) through which mass transfer are considered on the skin friction and heat transfer coefficients $(C_f(Re_x)^{1/2}, Nu(Re_x)^{-1/2})$. In the case of double slot suction ($A > 0$), the skin friction and heat transfer coefficients $(C_f(Re_x)^{1/2}, Nu(Re_x)^{-1/2})$ increase as the first slot begins and attain their maximum values before the trailing edge of the first slot. Next, $C_f(Re_x)^{1/2}$ and $Nu(Re_x)^{-1/2}$ decrease from their maximum values at the trailing edge of the first slot. Similar variations of skin friction and heat transfer coefficients $(C_f(Re_x)^{1/2}, Nu(Re_x)^{-1/2})$ are also observed in the second slot, and finally beyond the trailing edge of the second slot, $C_f(Re_x)^{1/2}$ and $Nu(Re_x)^{-1/2}$ remain finite in magnitude. Non-uniform double slot injection ($A < 0$) has the reverse effect as shown in Figs. 2 and 3. Moreover, the double slot suction ($A > 0$) and injection ($A < 0$) are, respectively, found to be more effective in increasing and decreasing both the skin friction and heat transfer coefficients $(C_f(Re_x)^{1/2}, Nu(Re_x)^{-1/2})$ as compared to the single slot suction ($A > 0$) and injection ($A < 0$). To be more specific, for $Ec = 0.1$ and $Pr = 0.7$ due to the increase of suction parameter A (>0) from 0.2 to 0.4, the increase of $C_f(Re_x)^{1/2}$ and $Nu(Re_x)^{-1/2}$ are approximately by 29 % and 23 %, respectively, at the middle of the first slot i.e., at $\xi = 1.0$ whereas the respective increase in $C_f(Re_x)^{1/2}$ and $Nu(Re_x)^{-1/2}$ are, approximately, by 47% and 36% at the middle of the second slot, i.e. at $\xi = 3.0$. Further more, it is noticed in Figs. 4 and 5 that the movement of double slot locations along the downstream direction has a significant effect on skin friction and heat transfer coefficients $(C_f(Re_x)^{1/2}, Nu(Re_x)^{-1/2})$ for the case of suction as well as injection.

4. Conclusions

Non-similar solution of a steady incompressible boundary layer flow over a slender cylinder with non-uniform double slot injection (suction) has been obtained from the origin of streamwise coordinate. Skin friction and heat transfer coefficients are found to be more effective due to non-uniform double slot injection(or suction) as compared with the effects of non-uniform single slot injection (or suction). Velocity and thermal boundary layer thicknesses increase with non-uniform slot injection but non-uniform slot suction reduces both velocity and thermal boundary layer thicknesses.

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